Margin of Error

ACTIVITY 39

Can't Get No Satisfaction

Lesson 39-1 Introduction to Margin of Error

Learning Targets:

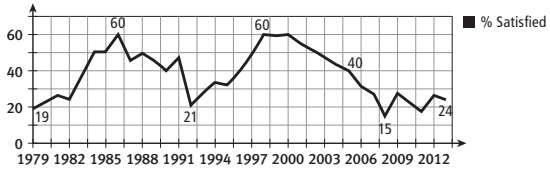
- Use margin of error in an estimate of a population proportion.
- Use simulation models for random samples.

SUGGESTED LEARNING STRATEGIES: Predict and Confirm, Think Aloud, Debriefing, Discussion Groups

Since 1979, Gallup, a national polling organization, has reported survey results of the question, "In general, are you satisfied or dissatisfied with the way things are going in the United States at this time?"

The results from 1979 to 2012 are displayed on the graph shown.

Satisfaction With the Way Things Are Going in the U.S., Yearly Averages



2013 figure represents yearly average to date.

1. Describe the meaning of the graph and characteristics that may be of interest to a person studying this graph.

The graph displays the proportion of Americans satisfied with the way things are going in the U.S. by year. Maximum and minimum points on the graph may be of interest, as they could indicate overall impressions of the country's state of being.

2. From 2000 to 2012, there is a steady decline in the satisfaction proportion. What historical events may account for such a decline?

The events of 9/11/2001, wars in Afghanistan and Iraq, the economic collapse of 2008, congressional gridlock, and other possible factors may have contributed to the decrease.

My Notes

Common Core State Standards for Activity 39

HSS-ID.A.1 Represent data with plots on the real number line (dot plots, histograms, and box plots).

HSS-ID.A.4 Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets, and tables to estimate areas under the normal curve.

ACTIVITY 39

Investigative

Activity Standards Focus

In Activity 39, students investigate how to use a margin of error in an estimate of a population proportion.

Students will use simulation models for random samples.

Lesson 39-1

PLAN

Pacing: 1 class period

Chunking the Lesson

#1–2 #3–7

#8–9 #10–13

Check Your Understanding

Lesson Practice

TEACH

Bell-Ringer Activity

Print out examples of line graphs from the internet for small groups of students to interpret. Have students describe the meaning of a given graph and its characteristics that may be of interest to others.

Developing Math Language

This activity contains the vocabulary words *margin of error* and *sample proportion*. Have students add the terms to their math notebooks, and encourage them to include an example of each term.

Use the class Word Wall to keep the new terms in front of students as they work on this activity.

1–2 Predict and Confirm Have students interpret the line graph. Point out that the *population proportion* is shown as a percent. A population proportion is part of a population with a particular attribute, in this case satisfaction.

3 Marking the Text, Think Aloud
Be sure that students understand the situation described in the first paragraph. Have them highlight *20% of Americans* and extend the graph on the previous page to reflect this result of the random survey for 2013. Have students mark the text that follows Item 3. Note that statisticians use a 95% confidence as the standard for a reliable survey.

Differentiating Instruction

Support students who do not understand the meaning of a *random sample,* a sample in which every element in the population has an equal chance of being selected. Have students describe possible populations that would not be random for this survey (populations of a specific gender, age, or income).

4–5 Think Aloud Ask students to determine how many satisfied people we would expect from a random sample of 10 people for the year 2000. *[6 people]* Ask, "Why do you think it was so high?" *[The new millenium created a sense of optimism.]*

My Notes

The results of the 2013 Gallup poll asking this question, conducted on November 7–10, 2013, indicated that 20% of Americans are satisfied with the way things are going in the United States. These results were based on telephone interviews with a random sample of 1039 adults, aged 18 and older, living in all 50 U.S. states and the District of Columbia.

3. Why did the Gallup pollsters use a random sample to establish this proportion of satisfied Americans?
It would be unreasonable to assume that Gallup could poll the entire American population of over 300,000,000 people in a timely manner.

Random samples are frequently used to make inferences about entire populations. Since the samples chosen are random and rely on chance, the laws of probability allow us to determine how sample results compare to an actual population proportion. The Gallup poll description continues with the following statement: "One can say with 95% confidence that the margin of sampling error is ±4 percentage points."

4. What is the meaning of this statement with respect to the fact that 20% of the Americans polled stated that they were satisfied with the way things were going in the United States?
It means that we are 95% confident that the actual percentage of Americans who are satisfied is between 16% (20% − 4%) and 24% (20% + 4%).

MATH TERMS

The **margin of error** indicates how close the actual proportion is to the estimate of the proportion found in a survey of a random sample.

The phrase "±4 percentage points" in the statement is called the ***margin of error***. Random samples have characteristics that set bounds on the errors that are likely to exist in the results of that random sample. In this activity, we will investigate these characteristics.

5. The Gallup poll indicated that 20% of the population was satisfied with how things were going in the United States in November 2013. If the actual population proportion is 20%, how many satisfied people would you expect from a random sample of ten people?
20% of 10 is 2; therefore, you would expect two of the ten people to be satisfied.

Lesson 39-1
Introduction to Margin of Error

My Notes

6. It is possible that your random sample of ten people in Item 5 could yield results that differ from your answer to Item 5. Which results would not be surprising? Which results would be surprising?
Even though two of ten is expected in Item 5, it would not be surprising to have results of zero, one, three, or four people indicating satisfaction. However, it would be surprising if the results indicated that five or more people in the random sample of ten were dissatisfied.

7. Given the actual population proportion is 20%, how many satisfied people would you expect from a random sample of 100 people? How different from your expected value must a result be for it to be a "surprising" result?
Answers will vary, but responses should be in an interval with 20 roughly in the middle. A common answer will be 10–30 people. Students may have an intuitive, yet not clearly defined, idea of how far away from 20 a result would have to be to be considered "surprising."

Using your graphing calculator, you can perform a simulation for the situations in Items 5 and 7 to model the selection of a random sample and the number of "successes" in that sample.

8. Use the *randBin(* function of your calculator to perform ten different simulations of the survey in Item 5. How many satisfied people exist in a random sample of ten people if the actual proportion is 0.20? Does your result agree with your answer to Item 6?
Students will use the *randBin(10, 0.2)* command and repeat it ten times or use *randBin(10, 0.2,10)* to perform all ten simulations at the same time. A sample response using this function is 3, 2, 1, 1, 1, 3, 4, 2, 0, 2. Results will be reasonably similar to Item 5.

9. Compare the result of your imaginary survey with the ones conducted by the others in your group. Explain why the results are likely different from one another.
Random samples of people will not all be the same, nor will they be equally representative of the population.

MATH TIP

To perform a simulation of a survey, generate imaginary data based on assumptions about actual population characteristics.

TECHNOLOGY TIP

To find the *randBin* function on the TI-84, press MATH and the arrow keys to select the *PRB* menu, and select *randBin(*. The first entry is the number of subjects in the random sample, followed by a comma, and then the probability of "success" for each subject in that random sample. Press ENTER and the result is the number of "successes" for one random sample. If you would like to perform the simulation a number of times, you can follow the probability with a comma, followed by the number of simulations you would like to perform.

For example, to find the number of successes in one random sample of ten people with a probability of success of 0.5, enter *randBin(10, 0.5)* To find the number of successes in eight such random samples, enter *randBin(10, 0.5, 8)*.

ACTIVITY 39 Continued

6–7 Think-Pair-Share Students may have an intuitive, yet not clearly defined, idea of how far away a result would have to be in order to be considered "surprising."

8–9 Sharing and Responding Have students work in small groups to use the *randBin* function on the TI-84 to create ten simulations of the survey in Item 5. Be sure that each student has an opportunity to use the calculator function. Students can use the *randBin(10, 0.2)* command and repeat it ten times, or use *randBin(10, 0.2,10)* to perform all ten simulations at the same time.

For additional technology resources, visit SpringBoard Digital.

10 Sharing and Responding, Group Presentation Have students work in groups of four, with each student performing a specific task. One student can calculate the sample proportion for each result of the random survey for Item 5; another student can create the histogram that displays the distribution of the proportions; another student can calculate the mean, and; a fourth student can calculate the standard deviation. Have students in the group collaborate to present a summary of the results for the class.

Teacher to Teacher

Directions for using Excel to create a histogram can be found online.

Differentiating Instruction

Support students who do not know how to calculate the *mean* and the *standard deviation*. Have students work in pairs to review these skills. Check students' work.

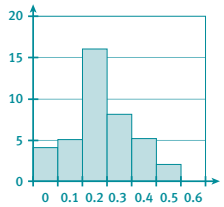
11–12 Sharing and Responding, Group Presentation Students repeat the processes and activities of the previous items using the results of the random survey for Item 7.

My Notes

10. Since the survey results are concerned with the proportion of people who are satisfied, convert each of your results into a *proportion*. The proportion for each result is called the ***sample proportion***. Combine the proportions from your surveys with the others in your group so that you have 40 survey results.

a. Create a histogram to display the distribution of proportions, and comment on the shape of your group's distribution.

Since there were ten subjects in the survey, dividing the survey results by ten will yield the proportion. An example of the proportions for the sample response in Item 8 is 0.3, 0.2, 0.1, 0.1, 0.1, 0.3, 0.4, 0.2, 0, 0.2. Combined with the three other groups for a total of 40 surveys, the histogram (choose intervals of 0.1) should look skewed right. An example is below.



b. Compute the mean and standard deviation for the 40 survey proportions.

The mean will be near 0.2 (for this sample, the mean is 0.2275) and the standard deviation will be relatively large (for this sample of 40 survey results, it is 0.128).

11. Use the *randBin(* function of your calculator to perform ten different simulations of the survey in Item 7. How many satisfied people exist in a random sample of 100 people if the actual proportion is 0.20? Does your result agree with your answer to Item 7?

Students will use the *randBin(100, 0.2)* command and repeat it ten times, or use *randBin(100, 0.2,10)* to perform all ten simulations at the same time. A sample response using this function is 14, 24, 15, 16, 16, 21, 19, 18, 21, 21. Results should be reasonably similar to your answer for Item 7.

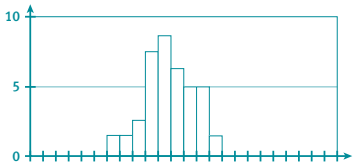
Lesson 39-1
Introduction to Margin of Error

My Notes

12. Combine the results of your survey with the others in your group so that you have 40 survey results, and find the *proportion* of satisfied subjects for each survey.

a. Create a histogram to display the distribution of proportions, and comment on the shape of your group's distribution.

A sample histogram is displayed below. For these proportions, interval widths will differ from group to group, but a width of 0.02 may provide good results. Regardless of interval widths, however, students should recognize that the shape of the distribution is approximately normal.



b. Compute the mean and standard deviation for the 40 survey proportions.

The mean will be near 0.2 (for this sample the mean is 0.211), and the standard deviation will be relatively small (for this sample of 40 survey results, it is 0.040).

13. Compare and contrast the means and standard deviations for the combined surveys of ten subjects and for the combined surveys of 100 subjects. What conclusion can you infer from these results?

The means for the surveys of 10 and 100 subjects are reasonably close to 0.2, with little difference between them. The standard deviation for the surveys of 10 subjects is much larger than the standard deviation for the surveys of 100 subjects. Students can reasonably conclude that the larger random sample results in less variation of the sample proportion. The concept of less variation with a larger sample is very important in determining margin of error in Lesson 39-2.

ACTIVITY 39 Continued

13 Graphic Organizer Have students create their own graphic organizer to compare and contrast the combined surveys of 10 subjects and of 100 subjects. Intuitively, students should realize that a larger sample size will result in a smaller variation.

Differentiating Instruction

Some students may enjoy repeating these simulations using larger sample sizes. Have them work individually or in groups to create a report.

Check Your Understanding

Debrief students' answers to these items to ensure that if they understand how to interpret the results of random surveys.

Answers

14. The results indicate that the poll's author is 90% confident that the actual proportion of the city's voters who support the mayor is between 47% and 59%.

15. Answers will vary, but simulation descriptions should state that two of the six possible outcomes on the number cube will represent success, and the other four outcomes will represent failure. Students should expect 4 successes from 12 trials. Results of the 12 trials will vary.

ASSESS

Students' answers to Lesson Practice problems will provide you with a formative assessment of their understanding of the lesson concepts and their ability to apply their learning.

See the Activity Practice for additional problems for this lesson. You may assign the problems here or use them as a culmination for the activity.

LESSON 39-1 PRACTICE

16. No; the survey shows that with 95% confidence, the actual proportion of students who scored above average is between 44% and 52%. Most of this range is at or below 50%. Jorge's claim of "most" implies a percentage greater than 50%.

17. Students could use the TI-84 *randBin*(*100, 0.54*) function 20 times or *randBin*(*100, 0.54, 20*) once. For a random digits table, students could assign digits 00-53 as successes and 54-99 as failures. Then they would begin on a line in the random digits table and choose the first 100 non-repeating two-digit clusters to represent the 100 subjects and count the number of successes. This process would then be repeated 20 times.

18. Answers will vary. For a sample generated with a TI-84, the mean is 53.7 and the standard deviation is 4.378.

ADAPT

Check students' answers to the Lesson Practice to ensure that they can analyze the results of surveys. Provide additional practice as needed.

My Notes

Check Your Understanding

14. In the days prior to a mayoral election, a poll reported, with 90% confidence, that the current mayor had support of 53% of the city's voting population, with a margin of error of 6%. Write a sentence to interpret the results of the survey.

15. Describe a procedure that uses a number cube to simulate a population proportion of 33%. How many successes would you expect from 12 trials? Perform the simulation 12 times, record your results, and compare them to your expectations.

LESSON 39-1 PRACTICE

16. Jorge claimed that the results of a survey supported his claim that most of the students in the junior class scored above average on the PSAT test. Valentina read the results of the survey to Jorge: "A survey of a simple random sample of students in the junior class indicated that 48% of them scored above average on the PSAT test. One can say with 95% confidence that the margin of error for this survey is plus or minus 4%." Is Jorge correct that the survey supported his claim?

The Gallup-Healthways Well-Being Index tracks, on a daily basis, the proportion of Americans who say they experienced happiness and enjoyment without stress and worry on the previous day. On one particular day, the survey of 500 people indicated that 54% were happy, with a margin of error of ± 5%.

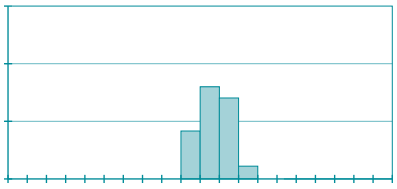
17. Using technology or a random digits table, describe how you could simulate 20 repetitions of such a survey for a random sample of size 100.

18. Perform the simulation that you described in Item 15, and find the mean and standard deviation.

19. Change your results to proportions and display them on a histogram. Use an interval width of 0.1.

20. Describe the shape of your distribution. Identify proportions that you would expect in such a simulation, and identify proportions that would be surprising in such a simulation.

19.



Answers will vary; a sample histogram is shown.

20. Distributions will likely be normal in shape. Unexpected results may be those more than two distributions away (less than 0.45 or greater than 0.63).

My Notes

Learning Targets:

- Use margin of error in an estimate of a population proportion.
- Relate margin of error to the population proportion and to the sample size.

SUGGESTED LEARNING STRATEGIES: Predict and Confirm, Think Aloud, Debriefing, Discussion Groups

"In general, are you satisfied or dissatisfied with the way things are going in the United States at this time?" For this question of interest, recall that the Gallup organization reported that for results based on this sample of 1039 adults, you can say with 95% confidence that the margin of error is ±4 percentage points.

The distribution of proportions of those who indicate they are satisfied for all possible samples of size n from the population is called the ***sampling distribution*** of the population for that statistic.

1. What is the population for this question of interest? Why is it not feasible to find the sampling distribution of size $n = 1039$ for this population?

   The population for the question of interest is all adults living in the United States. Answers will vary for the feasibility question, but students will recognize that it is difficult to create all possible samples of 1039 adults from an adult population that exceeds 240,000,000 (U.S. Census Bureau estimates that 76.5% of the U.S. population of 314,000,000 is 18 years or older, about 240,000,000 adults).

While it is not possible to find the sampling distribution for this statistic, you did generate some ideas by finding a large number of samples using simulations in the previous lesson.

2. In Items 10 and 12 from Lesson 39-1, which distribution was approximately normal? What were the sample sizes in those distributions?

   The distribution in Item 12 with $n = 100$ was approximately normal, while the distribution in Item 10 with $n = 10$ was skewed right.

Lesson 39-2

PLAN

Pacing: 2 class periods

Chunking the Lesson

#1–2	#3–6
Example A	#7–13
Check Your Understanding	
Lesson Practice	

TEACH

Bell-Ringer Activity

Have students work in small groups to describe the three different types of distributions—skewed left, normal, and skewed right—and draw and label a graph to illustrate each type.

Developing Math Language

This lesson presents vocabulary words that describe conditions for statistical inference. After students have completed this lesson, have them create a Vocabulary Organizer for the math terms as well as the statistical techniques used. Exact definitions of the math terms may be difficult for some students to articulate. However, most students will have an intuitive understanding of the concepts as they work through the lesson.

1–2 Activate Prior Knowledge, Predict and Confirm Discuss the idea that statisticians look at random samples of a population to get an idea of the characteristics of the entire population when the population is very large.

The characteristic we are focusing on is the distribution of the population. The distribution of the sample or *sampling distribution* can be described by the shape of the histogram.

3–4 Close Reading, Marking the Text How can we determine if the distribution of a population is normal without having to look at the histograms of increasingly larger simulations of random samples? Statisticians can allow a distribution to be considered approximately normal if the two conditions

- $n(p) \geq 10$
- $n(1 - p) \geq 10$

are satisfied. These two conditions are called the *normal conditions*. Have students place the term and the conditions on the class Word Wall and in their math notebooks.

For Item 3, n is the original sample size, 1039 people. Both the original sample size and a sample of 100 satisfy the normal condition.

Note that for a population with *any* distribution, the distribution of the sample approaches a *normal* distribution as the sample size increases. This allows for making inferences about the entire population.

My Notes

As sample sizes increase, the sampling distribution becomes more and more normal. If a random sample of size n has a proportion of successes p, there are two conditions that, if satisfied, allow the distribution to be considered approximately normal. Those two conditions are $\boldsymbol{n(p) \geq 10}$ and $\boldsymbol{n(1 - p) \geq 10}$.

3. Show that Gallup's survey meets the normal conditions.

$n = 1039$ and $p = 0.20$, so $np = 1039(0.2) = 207.8 > 10$ and $n(1 - p) = 1039(0.8) = 831.2 > 10$. Therefore, Gallup's survey meets the normal conditions.

4. Show that the simulation performed with $n = 10$ *does not meet* the normal condition and that the simulation performed with $n = 100$ *does meet* the normal condition.

$n = 10$ and $p = 0.20$, so $np = 10(0.2) = 2$, and 2 is NOT greater than or equal to 10. Likewise, $n(1 - p) = 10(0.8) = 8$, and 8 is NOT greater than or equal to 10. Therefore, the normal condition is not satisfied.
$n = 100$ and $p = 0.20$, so $np = 100(0.2) = 20 > 10$ and $n(1 - p) = 100(0.8) = 80 > 10$. Therefore, for $n = 100$, the normal condition is satisfied.

Lesson 39-2
Computing Margin of Error

My Notes

In general, when investigating a question of interest, you are not aware of the actual population statistic. However, by taking a simple random sample of an appropriate size, you can make inferences about the entire population. Also recall that normal distributions are completely described by two statistics: the mean and the standard deviation.

The standard deviation for a sampling distribution is given by

$$\sqrt{\frac{p(1-p)}{n}}$$

5. What is the meaning of the standard deviation with respect to a sample proportion?

It represents the average distance of a sample proportion from the actual proportion. It's the spread or variation from the actual population proportion.

6. In your simulations, you used $p = 0.20$ and $n = 100$. To be more accurate, would you prefer to use $n = 1000$? Use the formula to evaluate standard deviations to support your answer.

When $n = 100$, the standard deviation is 0.04, and when $n = 1000$, the standard deviation is 0.013. To increase accuracy, we prefer to use $n = 1000$ since it has less variation from the actual proportion.

MATH TIP

In the previous lesson, you discovered that the mean of the proportions of your sampling distributions was very close to the actual proportion. This is because the mean of the proportions of the entire sampling distribution is equal to the actual proportion. Therefore, *we can consider the proportion p of the random sample as the actual proportion.*

ACTIVITY 39 Continued

5–6 Activating Prior Knowledge, Summarizing Review the term *standard deviation*. Write the formula for the standard deviation for a sampling distribution on the class Word Wall. Have students include this in their math notebooks.

Ask students to give a brief statement of the main point in this section using a sample size of 100 and a sample size of 1000.

Example Shared Reading Students are to verify the information the city government provided with regard to a survey about recycling pickup. Have students follow each of the bulleted steps. Encourage them to work in small groups. Be sure that each student understands how to use technology to find the statistical measures needed.

Differentiating Instruction

Support students who do not know how to find the z-score from a table or using a calculator. Have students work in pairs. One student can find the negative z-score, while the other student can find the positive z-score.

My Notes

MATH TERMS

A **critical value** for an approximately normal distribution is the z-score that corresponds to a level of confidence.

TECHNOLOGY TIP

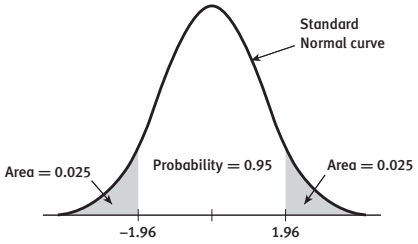
You may also use *invNorm*(0.25,0,1) on the TI- 84 to find the critical value. Use the mean 0 and standard deviation of 1 in this function because you are assuming that the values are standardized.

The Gallup survey stated that "the margin of error is ±4 percentage points." The margin of error is the range about the sample proportion in which you would expect to find the actual population proportion. The margin of error is found by multiplying the standard deviation by the ***critical value***.

Example A

A city government said that, based on a survey of a random sample of 800 adults in the city, you can say that 25% of them prefer weekly recycling pickup, with 95% confidence that the margin of error is ±3 percentage points.

- The sample proportion is 0.25.
- Since $np > 10$, $800(0.25) = 200 > 10$ and $n(1 - p) > 10$, $800(1 - 0.25) = 600 > 10$, we can assume that the sampling distribution is approximately normal.
- You would like to be 95% confident in the statement; this will determine the critical value. Since the distribution is approximately normal, we can use the z-table or invNorm function on our calculators. Notice that the 95% interval is evenly divided on either side of our sample proportion (mean).



- Find 0.975 in the body of the z-table for the positive critical value (1.96) or 0.025 in the body of the table for the negative critical value (−1.96).
- Multiply ± 1.96 by the standard deviation,

$$\pm 1.96 \sqrt{\frac{0.25(1-0.25)}{800}} \approx \pm 1.96(0.0153) \approx 0.030$$

- 0.030 is the margin of error.
- Therefore, you are 95% confident that the actual proportion of city residents that prefer weekly recycling pickup is 25% with a margin of error of ± 3%.

7–13 Discussion Groups Debrief these items by assigning different groups the responsibility of reporting their solutions on whiteboards to the class. In this way, students can compare and contrast their answers with those of other groups.

Lesson 39-2
Computing Margin of Error

My Notes

Victor, a member of the Student Government Association at his high school, wanted to know if students approved of the theme of the school's homecoming dance. He polled a simple random sample of 120 subjects from the population of 2000 students at his school, and 72 of the responses indicated approval. Victor would like to report back to the SGA with 90% confidence in the results of his survey.

7. What is the sample proportion that indicated approval?

$\frac{72}{120} = 0.6$, so the sample proportion is 0.6.

8. Victor assumes that the sampling distribution for his poll is approximately normal. Show that he is correct in his assumption.

$120(0.6) = 72 > 10$ and $120(1 - 0.6) = 48 > 10$, therefore Victor can assume the sampling distribution is approximately normal.

9. Victor wants to report with 90% confidence in his results.

a. On a normal distribution with 90% evenly divided on either side of the sample proportion (mean), what two probability values would you want to identify?

Since Victor wants 90% confidence, there will be 10% to divide evenly between the two ends of the normal distribution. Therefore, the two probability values will be 0.05 and 0.95.

b. What are the critical values associated with these probabilities?

From the *z*-table, values of ± 1.64 or ± 1.65 are acceptable.

(0.95 is exactly between 0.9495, for a *z*-score of 1.64, and 0.9505, for a *z*-score of 1.65; symmetry assures the same result, but negative, for 0.05.) Using the *invNorm(0.95, 0, 1)* function, the *z*-score to three decimal places is 1.645.

10. What is Victor's margin of error?

$\pm 1.645\sqrt{\frac{0.6(1-0.6)}{120}} \approx 0.074$; therefore, his margin of error is $\pm 7.4\%$.

7–13 (continued) Provide students the opportunity to discuss their results for these items in their small groups first. As student groups share their results with the class, encourage and facilitate meaningful discussion about the methods of finding the statistics and students' interpretations of them.

My Notes

11. Write a sentence that reports Victor's results to the Student Government Association at his school.

Victor will report, "I am 90% confident that the actual proportion of students that approved of the theme of the homecoming dance is 60%, plus or minus a margin of error of 7.3%." He might also say, " I am 90% confident that the actual proportion of students that approved is between (60% − 7.3% =) 52.7% and (60% + 7.3% =) 67.3%."

12. Without performing the computations, how do you think the margin of error would change if the number of students that Victor polled were 80? How do you think it would change if the number of students were 200?

Students will realize that as *n* increases, the margin of error decreases, since *n* is in the denominator. Therefore, the margin of error for 80 will be less than 7.3%, and the margin of error for 200 will be less than the margin of error for 80.

13. Compute the actual margin of error for $n = 80$ and $n = 200$ to confirm or revise your answer to Item 7.

For $n = 80$, the margin of error is $\pm 9\%$; for $n = 200$, the margin of error is $\pm 5.7\%$.

Lesson 39-2
Computing Margin of Error

My Notes

Check Your Understanding

Recall that the standard deviation of a sample proportion is represented by $\sqrt{\frac{p(1-p)}{n}}$.

14. Describe the meaning of each variable. Explain what happens to the standard deviation when the value of n increases.

15. For a fixed value of n, what value of p would yield the largest standard deviation?

LESSON 39-2 PRACTICE

Sofia is a credit card specialist with a large financial institution. She is interested in knowing what proportion of the bank's credit card holders have credit scores in the good or excellent range (scores of 680 and above). Sofia surveyed a simple random sample of 1000 of the bank's credit card customers and found that 750 of them had credit scores of 680 and above.

16. For Sofia's survey, identify each of the following.
- **a.** the question of interest
- **b.** the population
- **c.** the sample proportion

17. Write the standard deviation for the sample proportion.

18. Sofia wants to be 98% confident in her estimate of the actual proportion. What critical values will she use in her determination of the margin of error?

19. Compute the margin of error, and write a sentence that describes the results of Sofia's survey.

Check Your Understanding

Debrief this lesson by having students examine the standard deviation formula and explain what happens as n varies for a fixed p and as p varies for a fixed n.

Answers

14. p is the sample proportion (the proportion of successes) for the random sample. n is the number of subjects in the random sample. As the value of n increases in the denominator, the value of the fraction becomes smaller, so for a fixed value of p, the standard deviation becomes smaller.

15. The largest standard deviation will occur when $p(1-p)$ is largest. This will occur at the maximum of the function $f(p) = p(1-p)$, a quadratic function whose vertex is at 0.5. Therefore, the largest standard deviation will occur when $p = 0.5$.

ASSESS

Students' answers to Lesson Practice problems will provide you with a formative assessment of their understanding of the lesson concepts and their ability to apply their learning.

See the Activity Practice for additional problems for this lesson. You may assign the problems here or use them as a culmination for the activity.

LESSON 39-2 PRACTICE

16. **a.** the proportion of customers with a credit score of 680 and above.
b. the bank's credit card customers.
c. $\frac{750}{1000} = 0.75$.

17. $\sqrt{\frac{0.75(1-0.75)}{1000}} \approx 0.014$

18. The z-scores that correspond to 0.99 and 0.01 are ± 2.33.

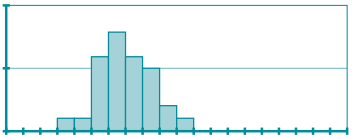
19. Sofia is 98% confident that 75% of the bank's credit card customers have credit scores in the good or excellent range with a margin of error of $\pm 3.2\%$. Alternatively, she may say that she is 98% confident that the actual proportion of the bank's credit card customers with a credit score of good or excellent is between 71.8% and 78.2%.

ADAPT

Check students' answers to the Lesson Practice to ensure that they can describe the results of a survey given the size of the sample and the number of successes. If students have difficulty, review the steps presented in the lesson.

ACTIVITY PRACTICE

1. 5 red and 15 green jellybeans would not be surprising because it is only one jellybean away from 30% red and 70% green.
2. 15 red and 5 green jellybeans are so different from the actual proportions of 30% red and 70% green that such a selection would be very unusual.
3. Assign numbers 0-2 to represent red jellybeans and 3-9 to represent green jellybeans. Choose a line on the random digits table and select the first 20 digits. Count the number of digits that correspond to red jellybeans. Repeat the process 30 times.
4. Use *randBin*(*20, 0.3*) 30 times or *randBin*(*20, 0.3, 30*) once to model the samples.
5. Answers will vary, but a sample simulation provided these results, which should be similar to students' work. The mean of the proportions for the thirty simulations is 0.325, very close to the 0.3 proportion of red jellybeans. Therefore, these results agree with Item 1.



A sample histogram shows an approximately normal distribution. The standard deviation is 0.079, which means that a proportion of 0.75 is very unlikely as it is more than five standard deviations from the mean. This agrees with Item 2.

6. The population consists of all American adults.
7. What is China's relationship with the United States?
8. The sample proportion is 0.76.
9. $\pm 1.96\sqrt{\frac{0.76(1-0.76)}{2007}} \approx \pm 0.019$
10. The margin of error would increase as the population is smaller.
11. $\pm 1.96\sqrt{\frac{0.76(1-0.76)}{1000}} \approx \pm 0.026$

ADDITIONAL PRACTICE

If students need more practice on the concepts in this activity, see the Teacher Resources at SpringBoard Digital for additional practice problems.

ACTIVITY 39 PRACTICE

Write your answers on notebook paper.
Show your work.

Lesson 39-1

A jar contains 1000 jellybeans that are colored either red or green. 30% of the jellybeans are red, and the remaining jellybeans are green. Assume that the jellybeans are well mixed and that a random sample of 20 jellybeans is chosen from the jar.

1. Would it be unusual to pull out five red jellybeans and 15 green jellybeans? Explain.
2. Would it be unusual to pull out 15 red jellybeans and five green jellybeans? Explain.
3. Describe a simulation that you could perform with a random digits table to model 30 such samples.
4. Describe a simulation that you could perform with a graphing calculator that would model 30 such samples.
5. Perform one of the simulations that you described in Item 3 and Item 4. Convert the number of jellybeans to proportions, and explain how the results of your simulation agree or disagree with your responses to Item 1 and Item 2.

Lesson 39-2

In late 2011 the Gallup organization surveyed a random sample of 2007 American adults and asked them what they thought about China's relationship with the United States. 76% of those surveyed said that China was either "friendly" or "an ally." Gallup reported the following statement along with the survey results: "For results based on the total sample size of 2007 adults, one can say with 95% confidence that the margin of error attributable to sampling and other random effects is ±2.68 percentage points."

6. What is the population?
7. What is the question of interest?
8. What is the sample proportion?
9. The margin of error reported uses some advanced statistical methods to adjust for sample and population characteristics. Find the "unadjusted" margin of error for this survey.
10. If the sample size for this survey were 1000, what changes would you expect in the margin of error?
11. Compute the margin of error for a sample size of 1000.

MATHEMATICAL PRACTICES

Construct Viable Arguments and Critique the Reasoning of Others

12. In 2011, a New York University professor of journalism, Charles Seife, wrote, "Random events behave predictably in aggregate even if they're not predictable individually." How does that principle relate to the concept of a margin of error in a survey result?

12. Answers will vary. Sample response: If you sample one person at random from a population and ask him or her a question, you cannot predict what he or she will say; the behavior is not predictable individually. But if you ask lots of randomly selected people the same question and determine the proportion of them who say "yes," you can be pretty sure that the proportion will be within a certain range (the margin of error) of the population proportion—the aggregate behavior is predictable. The responses become more predictable the more people you survey.